DOCUMENT RESUME

ED 117 411 CE 006 055

AUTHOR Charnes, A.; And Others
TITLE A Multi-Objective Model for Planning Equal Employment Opportunities. Research Report No. 23. Revised Edition.
INSTITUTION Office of Civilian Manpower Management (Navy), Washington, D.C.
SPONS AGENCY Office of Naval Research, Washington, D.C. Personnel and Training Research Programs Office.
REPORT NO OCMM-RR-23
PUB DATE Oct 75
NOTE 33p.

EDRS PRICE MF-$0.83 HC-$2.06 Plus Postage
DESCRIPTORS Administrative Organization; Employment Opportunities; *Equal Opportunities (Jobs); Linear Programing; *Manpower Development; *Models; Objectives; *Organizational Development; *Planning; Probability; Program Planning
IDENTIFIERS Markoff Processes

ABSTRACT

The document suggests some new modeling approcaches for equal employment opportunity (EEO) planning. Previous manpower planning models of the Office of Civilian Manpower Management have utilized multi-period Markoff processes embedded in goal programing (multiple objective) models. These are here extended to EEO plans directed to changing the mix of employees over time. At each point in the planning interval, the organization is taken as given (for example, in terms of the probabilities for promotion, transfer, etc.) when formulating manpower programs. Over time, however, these organization processes are submitted to planned changes which alter the probabilities of occurrence for these events. The Merit Promotion System is preserved and other controls are also imposed explicity for the exercise of managerial discretion. The focus here is on an ordinary (absolute value) formulation of objectives and a numerical illustration is supplied with differing weights for each of the indicated classes of objectives. Other types of objectives are briefly discussed, along with different approaches to problems of validation and, subsequently, implementation in a U. S. Navy context. (Author/EC)

OCMM RESEARCH REPORT NO. 23

A MULTI-OBJECTIVE MODEL FOR PLANNING EQUAL EMPLOYMENT OPPORTUNITIES

DEC 22 1975 CE

BY
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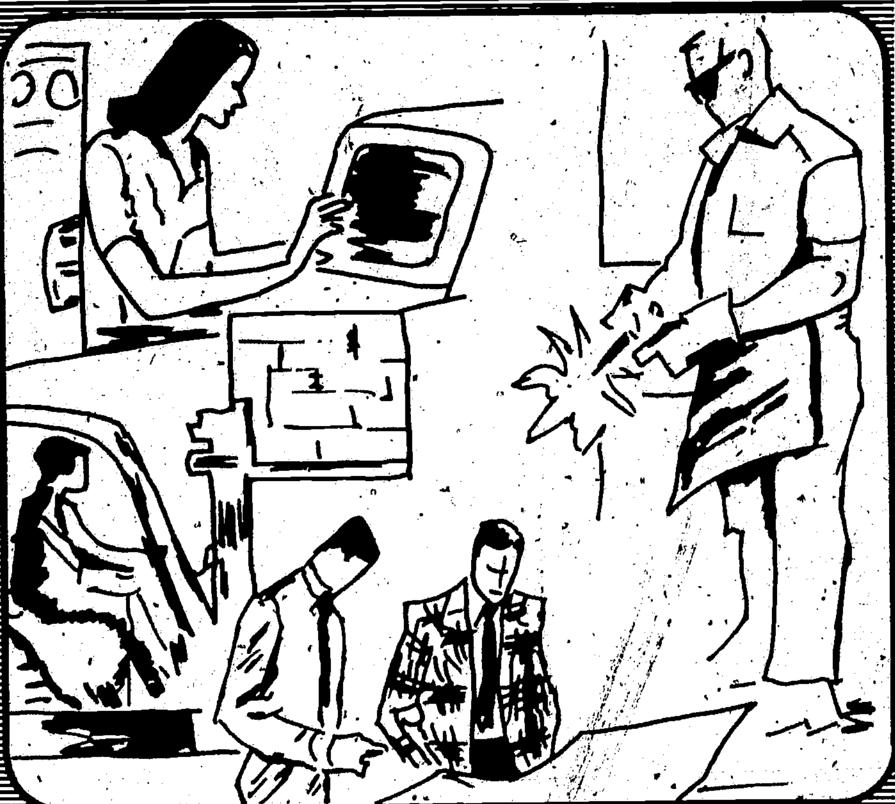

MANAGEMENT SCIENCES BRANCH · MANPOWER INFORMATION DIVISION
OFFICE OF CIVILIAN MANPOWER MANAGEMENT · DEPARTMENT OF THE NAVY

UNCLASSIFIED

SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

REPORT DOCUMENTATION PAGE

READ INSTRUCTIONS BEFORE COMPLETING FORM

1. REPORT NUMBER	2. GOVT ACCESSION NO.	3. RECIPIENT'S CATALOG NUMBER

4. TITLE (and Subtitle)
A Multi-objective Model for Planning Equal Employment Opportunities

5. TYPE OF REPORT & PERIOD COVERED
Technical Report
October 1975

6. PERFORMING ORG. REPORT NUMBER
OCMM Report No. 23

7. AUTHOR(s)
A. Charnes, W. W. Cooper, K. A. Lewis, R. J. Niehaus

8. CONTRACT OR GRANT NUMBER(s)
WR-6-0147

9. PERFORMING ORGANIZATION NAME AND ADDRESS
Office of Civilian Manpower Management,
Navy Department, Washington, D. C. 20390

10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS
62755N
RF 55-521; RF 55-521-801
NR 155-359

11. CONTROLLING OFFICE NAME AND ADDRESS
Personnel and Training Research Programs
Office of Naval Research (Code 458)
Arlington, VA 22217

12. REPORT DATE
October 1975

13. NUMBER OF PAGES

14. MONITORING AGENCY NAME & ADDRESS (if different from Controlling Office)

15. SECURITY CLASS. (of this report)
UNCLASSIFIED

15a. DECLASSIFICATION/DOWNGRADING SCHEDULE

16. DISTRIBUTION STATEMENT (of this Report)

Approved for public release, distribution unlimited

17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from Report)

18. SUPPLEMENTARY NOTES

Also released by Management Sciences Research Group, Graduate School of Industrial Administration, Carnegie-Mellon University, Report No. 373

19. KEY WORDS (Continue on reverse side if necessary and identify by block number)

Manpower Planning, Equal Employment Opportunity, Training and Assignment, Promotion and Transfer Probabilities, Budget Constraints, Salary Limitations, Goal Programming, Linear Programming, Multiple Objectives, Markoff Processes

20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

See Reverse Side

DD FORM 1 JAN 73 1473 EDITION OF 1 NOV 65 IS OBSOLETE
S/N 0102-014-6601

UNCLASSIFIED

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A Multi-Objective Model For Planning Equal Employment Opportunities

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This is a revised version of a paper presented at the meetings of the Institute of Management Sciences in Kyoto, Japan on July 25, 1975. It will be included in M. Zeleny, Ed. Multiple Criteria Decision Making: Kyoto 1975 (New York: Springer Verlag, 1975). The underlying research was sponsored by the Personnel and Training and Operations Research Programs of the Office of Naval Research. It was accomplished under ONR Work Request WR-6-0147 with the Office of Civilian Manpower Management, ONR Contract No. N-00014-75-C0616 with the University of Texas, and ONR Contract No. NR 760(24), NR 047-048 with Carnegie-Mellon University.

October 1975

OFFICE OF CIVILIAN MANPOWER MANAGEMENT
NAVY DEPARTMENT
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Abstract

Previous manpower planning models -- e.g., in the OCMM series -- have utilized multi-period Markoff processes embedded in goal programming (multiple objective) models. These are here extended to Equal Employment Opportunity plans directed to changing the mix of employees over time. At each point in the planning interval, the organization is taken as given, e.g., in terms of the probabilities for promotion, transfer, etc., when formulating manpower programs. Over time, however, these organization processes are submitted to planned changes which alter the probabilities of occurrence for these events. The Merit Promotion System is preserved and other controls are also imposed explicitly for the exercise of managerial discretion. The focus here is on an ordinary (absolute value) formulation of objectives and a numerical illustration is supplied with differing weights for each of the indicated classes of objectives. Other types of objectives are briefly discussed, along with different approaches to problems of validation and, subsequently, implementation in a U.S. Navy context.

1. Introduction

This paper suggests some new modeling approaches for EEO (equal employment opportunity) planning. On the one hand, contact is maintained with the main thrust of the OCMM series of models[1] in terms of prescribed goals approached via a goal programming (multiple time period) model with embedded Markoff processes. On the other hand the mixes involving minority and other personnel are changed by altering the Markoff elements so that, ultimately wanted mixes at all levels will be generated by associated changes in organization structure.

It is in the latter respect that this presentation differs from others, such as [5] and [7], which have proceeded either (a) from uses of Markoff processes to analyze present or projected conditions [5] or (b) from already developed goal programming formulations (with embedded Markoff processes) as in [7] which are also extended or altered for use in sequences of simulation types of approaches. Here again the latter route is also used except that we now view the problem from a two-fold standpoint as follows. First, the organization is viewed as effecting its recruitments and placements, etc., in order to meet its manpower goals "as closely as possible". Second, recruitment and placements are also effected in order to alter the Markoff elements. In this manner, longer range goals for altering the mixes of personnel -- minorities, females, etc. -- are also produced by changing present probabilities of promotion and transfer to new "steady state" values.

The processes to be considered are evidently factored into two parts which may be interpreted as follows. In one part, the first,

[1] See [2].

the organization is taken as given and is brought as close as possible to the goals that will enable it to do its job. In another, the second part, the organization structure is changed (via the indicated alterations in the Markoff elements) so that the desired mixes of personnel will automatically occur, just as they do now, but in altered proportions as a result of the changing mixes in recruitment and the resulting probability changes for promotion and transfer.

One way to deal with these problems is to consider a variety of different objectives which might be applied to each of the two (different) processes. For instance, discounting or compounding might be used, e.g., to bring possible future changes into a form that is comparable with the present. We are here dealing with social values, however, which are to be considered in a governmental type of organization. In particular this includes changes in social structure which, in turn, may bring such "costing" mechanisms (determined from present social structures) into question. This therefore poses questions which need to be resolved before such "present-value" approaches can be used. A use of standard compounding and discounting formulas is, of course, not the only possibility, but an inquiry into these alternatives involves a variety of additional considerations so that their study is probably best left for later (separate) attention. Also by setting this aside for later study we shall be able to proceed immediately and build upon a prior series of models developed at OCMM in terms of absolute value objectives which can then be reduced to equivalent linear programs.[2]. Hence, we will also then be able to study essential proper-

[2] See [1] and [2]

ties in such model developments via already available computer codes and also proceed toward implementation (e.g., planning by means of interactive computer arrangements), without undue delays in a program which already has some urgency about it.[1] The other, longer range research, such as is required for considering alternative objectives and functions can then be considered in later, or at least separate, contexts, but with a background of prior experience to guide it.[2]

The order for the presentations in this paper may now be summarized as follows. First the notational details and the personnel and organization transition constraints are set forth in the section that follows. These are then extended to allow for the adjunction of budgetary and other constraints which limit the amounts of recruiting, training and transfer that may be undertaken. After this has been done the objectives and the functional will be set forth and the whole model then collected in section 4. A numerical example will be given in section 5 and then a sketch of further courses for development and a summary provided in section 6, will conclude this article.

---

[1] See [4]

[2] Uses of constrained information theoretic functions for this purpose along with other such possibilities were discussed in the earlier draft of this paper distributed at the Kyoto meetings of TIMS.

2. Personnel and Organization Transition Constraints

The model will be developed in terms of generic categories in order to keep the presentation as simple as possible. The mode in which this may be expanded to keep explicit track of minority and non-minority groups in any detail that may be of interest will be clear, however. The concept of job categories will also be expanded beyond ordinary usage in order to include "training states" for specific "actual job" categories. That is, the preparatory period which is ordinarily needed to permit adjustment for <u>any</u> new job is here formalized into training periods which may be lengthened or shortened as might be required for personnel who may need more extended periods for this purpose. Finally, the current transition rates for promotion and transfer in the present organization will need to allow for additional flexible transfers within prescribed bounds.

To see how this is done we now proceed to develop the model details as follows. Let

(1) $x_i^k(t)$ = number of personnel of type k in job category i in period t

so that, e.g., we might designate the number of minority and non-minority males and females in category i at period t via

$$
(2)\quad
\begin{aligned}
x_i^1(t) &= \text{non-minority males}\\
x_i^2(t) &= \text{non-minority females}\\
x_i^3(t) &= \text{minority males}\\
x_i^4(t) &= \text{minority females}
\end{aligned}
$$

We shall choose the number for our job category indexes to reflect training status as well as actual job status via

$$(3)\quad i = \begin{cases} 1,3,5,\ldots,(2J-1) = \text{training status for the actual job category in the succeeding integer number} \\ 2,4,6,\ldots,2J = \text{actual job categories} \end{cases}$$

Thus, if i is an odd integer it will denote a training status for the actual job category of the succeeding (even) integer.

Next, we let

(4) m_{ij} = current or "historical" transition rate from category j to category i

and

(5) $z^k_{ij}(t)$ = number of type k in category j of period t additionally transferred to category i for period t.

I.e., $z^k_{ij}(t)$ represents an addition to the current organizational transfer rate that would otherwise be provided by m_{ij}.

Also, let

(6) $h^k_i(t)$ = number of type k hired from outside into category i in period t,

where $h^k_i(t) < 0$ represents a RIF (Reduction in Force) and $h^k_i(t) > 0$ represents an augmentation via outside recruitment into the organization. Evidently the additional (flexible) transfers out of type k category j cannot exceed the total number of this type in this category minus those who will leave the system, i.e.,

$$(7)\qquad \sum_i z^k_{ij}(t) \leq (\sum_i m_{ij})\, x^k_j(t-1)$$

Via "policy parameters" $f_{ij}^k(t) \geq 0$, we may also make the stipulations

(7.1) $$z_{ij}^k(t) \leq f_{ij}^k(\sum_{\ell} m_{\ell j})x_j^k(t-1) \text{ where } 0 \leq f_{ij}^k \leq 1.$$

Of course, we may also wish to require at least minimal proportions $p_i^k(t)$ of type k in category i during period t -- viz.,

(8) $$x_i^k(t) \geq p_i^k(t)x_i(t),$$

where

(8.1) $$x_i(t) = \sum_k x_i^k(t).$$

In allowing for additional personnel transitions to particular categories, it may be necessary to reduce their expected flows to other categories. We therefore let

(9) $y_{ij}^k(t)$ = number of type k, category j, not transiting to category i in period t via expected transition rate m_{ij}.

Thus, we require

(9.1) $$y_{ij}^k(t) \leq m_{ij}x_j^k(t-1),$$

and so that the additions can only come from subtractions (i.e., attrition, transfer or promotion),

(9.2) $$\sum_i \sum_j z_{ij}^k(t) = \sum_i \sum_j y_{ij}^k(t).$$

We shall refer to this as our "additive-subtractive balance" conditions.

Our basic equations of personnel change from period t-1 to t can now be written

$$(10) \qquad x_i^k(t) = \sum_j m_{ij} x_j^k(t-1) - \sum_j y_{ij}^k(t) + \sum_j z_{ij}^k(t) + h_i^k(t)$$

Rewriting this as

$$(10.1) \qquad x_i^k(t) = \sum_j \bar{m}_{ij}^k(t) x_j^k(t-1) + h_i^k(t)$$

where

$$(11) \qquad \bar{m}_{ij}^k(t) = m_{ij} + \frac{z_{ij}^k(t) - y_{ij}^k(t)}{x_j^k(t-1)},$$

we see that

$$(11.1) \qquad 0 \leq \bar{m}_{ij}^k(t) \leq m_{ij} + f_{ij}^k \leq 1.$$

3. Budget Constraints

In addition to the above we will generally need to consider budgetary constraints, which we here write as

(12.1) $\sum_i \sum_k c_i^1(t) x_i^k(t) \leq b^1(t)$

(12.2) $\sum_i \sum_j \sum_k c_{ij}^2(t) z_{ij}^k(t-1) \leq b^2(t)$

(12.3) $\sum_i \sum_k c_i^3(t) h_i^k(t) \leq b^3(t)$.

The first constraint represents the salary budget for all job occupants in period t. By assumption the same salary $c_i^1(t)$ is applicable to all occupants of job category i irrespective of their type. A similar assumption applies to the second constraint set where each $c_{ij}^2(t)$ represents transfer costs, e.g., salary plus training, for the flexible transfer from job type j to i. The third represents salaries plus recruiting costs for new hires (or penalties associated with RIF's when $h_i^k(t) < 0$) and, of course, $b^1(t)$, $b^2(t)$ and $b^3(t)$ represent budget limits for the activities in the constraints where they appear.

The above budget constraints will suffice, at least for the present, although they may need to be subsequently elaborated. For instance, it may be desirable to separate out the costs of meeting goals with a given organization structure and distinguish these from the costs of changing that structure -- and so on.

4. Functional and Objectives

We now formulate one possible functional and objective as

$$(13) \quad \text{minimize} \sum_{i,k,t} w_{ik}(t) \left| x_i^k(t) - g_i^k g_i(t) \right| + \sum_{i,t} w_i(t) \left| g_i(t) - \sum_k x_i^k(t) \right|$$

where the $w_i(t)$ and $w_{ik}(t) \geq 0$ are prescribed weights and the vertical strokes indicate that an absolute value is to be taken. Note that the $g_i(t)$ are the goals for numbers of personnel in job category i for period t, whereas the g_i^k are selected to conform to longer run goals for the proportions of type k personnel. The weights $w_i(t)$ will usually be greater than the $w_{ik}(t)$ in order to reflect the priority of filling the needs of the job first followed by attempting next to reach the goals for the proportions -- this may be regarded as a viability condition as we have already indicated, since the organization must first do its assigned jobs adequately if it is to survive at all.

We now collect all of the developments to this point in order to present and describe the complete model as follows:

$$(14.1) \quad \text{minimize} \sum_{i,k,t} w_{ik}(t) \left(\delta_{ik}^{+}(t) + \delta_{ik}^{-}(t)\right) + \sum_{i,t} w_i(t) \left(\delta_i^{+}(t) + \delta_i^{-}(t)\right)$$

subject to:

(14.2)

Goal Constraints
$$\begin{cases} \delta^+_{ik}(t) - \delta^-_{ik}(t) + x^k_i(t) = g_i(t)\,g^i_i \\ \delta^+_i(t) - \delta^-_i(t) + \sum\limits_k x^k_i(t) = g_i(t) \end{cases}$$

Transition Conditions
$$\left\{ -h^k_i(t) - \sum_j z^k_{ij}(t) - \sum_j m_{ij}\,x^k_j(t-1) + x^k_i(t) + \sum_j y^k_{ij}(t) = 0 \right.$$

Max. Additive Flexibility
$$\left\{ -z^k_{ji}(t) + f^k_{ji}\Big(\sum_\ell m_{\ell i}\Big)x^k_i(t-1) \ge 0 \right.$$

Max. Subtractive Flexibility
$$\left\{ -y^k_{ji}(t) + m_{ji}\,x^k_i(t) \ge 0 \right.$$

Additive-Subtractive Balance Conditions
$$\left\{ \sum_{ij}\sum z^k_{ij}(t) - \sum_{ij}\sum y^k_{ij}(t) = 0 \right.$$

Minimum EEO Proportions
$$\left\{ x^k_i(t) - p^k_i(t)\,g_i(t) \ge 0 \right.$$

Budgetary Constraints
$$\begin{cases} -\sum\limits_{ik}\sum c^1_i(t)\; x^k_i(t) \ge -b^1(t) \\ -\sum\limits_{ijk}\sum\sum c^2_{ij}(t)\; z^k_{ij}(t-1) \ge -b^2(t) \\ -\sum\limits_{ik}\sum c^3_i(t)\; h^k_i(t) \ge -b^3(t) \end{cases}$$

where $\delta^+_{ik}(t)$, $\delta^-_{ik}(t)$, $\delta^+_i(t)$, $\delta^-_i(t)$, $x^k_i(t)$, $z^k_{ij}(t)$, $x^k_{ij}(t)$ are non-negative for all i, j, k, t.

As can now be seen, this will lead to models involving large numbers of constraints and variables and hence will need further attention for practical applications. Other possibilities also exist, however, and these too will need to be explored via, e.g., the kinds of developments noted in the introduction to this paper which we now develop as follows. In any event, since the present paper is only intended to open possible patterns for future explorations we may now proceed to a numerical example, at least for purposes of illustration, as we do in the next section.

5. A Numerical Illustration

To study the workings of this model let us consider a numerical example, the details of which appear in Figure 1, with simplifying assumptions to hold down its size for purposes of illustration. Let there be two personnel types, minority and non-minority; two time periods, the first beginning when $t = 0$ and ending when $t = 1$, and the second beginning when $t = 1$ and ending when $t = 2$; and three job categories or levels so that category 1 = low job level, category 2 = training, and category 3 = high job level.

"Relative priorities" as given in Figure 1 are reflected in the weights α and β which are assigned to the functional. For this illustration we employ the values $\alpha = (1,1,1)$ and $\beta = (2,2,2)$.

The TOTAL MANPOWER GOALS, as referenced in the first set of constraints, are short-term in nature. They deal with satisfying the immediate operating needs of the organization by reference to the goals stipulated for the numbers of personnel in each job level for each time period. Each equation in this constraint set concentrates on one job level. Thus, as shown by the elements of these identify matrices, I, the on board minority personnel in a particular job category are added to the non-minority personnel on board at that same job level in relation to the indicated goal. In our conventions the resulting summation applies to the number of each at the end of the applicable period. To each sum, then, a deviational term is also added in order to reflect the numerical value by which the stated goal fails to ~~be~~ met, either positively or negatively. It is to these deviations that the weights are assigned so that minimizing objectives will push the solution toward meeting these goals with the indicated weights, or priorities, as close as the constraints will allow.

MATRIX DETAILS OF EQUAL EMPLOYMENT OPPORTUNITY MODEL

	ON-BOARD (MINORITY)	ON-BOARD (NON MINORITY)	POS. TOT. MAN. DEV.	NEG. TOT. MAN. DEV.	POS. PROP. DEV.	NEG. PROP. DEV.	ADD. PERS ADJUSTMENTS	SUB. PERS ADJUSTMENTS	HIRE	FIRES	SIGN	RHS
RELATIVE PRIORITIES			α α	α α	β β	β β						
TOTAL MANPOWER GOAL	I I	I I	-I -I	I I							= =	MAN. REQS.
PROPORTIONAL EEO GOALS	I I				-I -I	I I					= =	PROP REQS.
TRANSIT CONDITIONS	I -M I -M I						 -T* -T	 T T	 -I -I	 I I	= = =	INIT. POP. 0 0
		I -M I -M I					 -T -T	 T T	 -I -I	 I I	= = =	INIT. POP. 0 0
MAX. ADDITIVE FLEXIBILITY	f f	f f					-S+ -S	S S			≥ ≥	0 0
MAX. SUBTRACTIVE FLEXIBILITY	P P	P P						-S -S			≥ ≥	0 0
BALANCING CONDITIONS							e^T e^T	$-e^T$ $-e^T$			= =	0 0
MIN. EEO PROPORTIONS	I I										≥ ≥	MIN. PROP.
BUDGET CONSTRAINTS	c^1e^T c^1e^T	c^1e^T c^1e^T									≤ ≤	SALARY BUDGET
							c^2e^T c^2e^T				≤ ≤	TRANS COSTS
									c^3e^T c^3e^T	c^3e^T c^3e^T	≤ ≤	HIRE/FIRE COSTS

*T'S REPRESENT MATRICES OF 1'S OR -1'S AS APPROPRIATE FOR CATEGORIES IN WHICH TRANSITIONS CAN BE MADE

+S'S REPRESENT MATRICES OF 1'S OR -1'S AS APPROPRIATE FOR CATEGORIES IN WHICH FLEXIBILITIES OCCUR

FIGURE 1

The longer-run issue of changing mix of personnel is considered in the set of constraints called the PROPORTIONAL EEO GOALS. These longer-run considerations involve setting target values for the fractional part, i.e., the g_i^k from the functional, which are the desired steady-state proportions, that will yield the total number of personnel at each job level who are to be of minority status. In this illustration, the fractional parts to be used are displayed as in Figure 2.[1]

Figure 2

EEO PROPORTIONS

	JOB LEVEL 1 = LOW	JOB LEVEL 2 = TRAINING	JOB LEVEL 3 = HIGH
MINORITY STATUS	.2	.2	.2

Again, a set of weights are assigned to deviations from the targeted values, with, in this case, $\beta = (2,2,2)$ serving to drive solutions toward these proportional goals via the indicated minimization.

Over a period of time "on-board" personnel may move from one job category to another. Historically, such movements can be catalogued and their probabilities determined.[2] These comprise the set of expected transition rates which are represented as the matrices M (with entries m_{ij}) in Figure 1. Specific values for these expected transition rates are provided in Figure 3. Thus, as a result of past and current data, for example, we expect 75% of on-board personnel in job category 1 at the start of a time period to remain at that (low) job level at

[1] In this case we are dealing with only one minority for which a proportion must be defined, and hence, there is only one row in this display.

[2] See [2].

the end of the time period. Alternatively 10% (i.e., 1/10) of all personnel on board in job category 1 at the beginning of the time period are expected to move to a training position (job level 2) by the end of that period, while 5% of those at job level 1 will move to job category 3. Where zero values appear in this matrix no transition is expected, so, from Figure 3, we can see that we do not expect any on-board trainees at the beginning of a time period to remain as trainees at the period's end. (See row 2, column 2.)

Figure 3

EXPECTED TRANSITION RATES

TO \ FROM	JOB LEVEL 1=LOW	JOB LEVEL 2=TRAINING	JOB LEVEL 3=HIGH
JOB LEVEL 1=LOW	.75	.2	0
JOB LEVEL 2=TRAINING	.1	0	0
JOB LEVEL 3=HIGH	.05	.7	.9

If we sum the probabilities down any column (e.g. .75 + .1 + .05 = .90), we will be calculating the probability that those who were on-board in some capacity will still be on board at the end of the period. That is, we follow the same convention as in [2] to allow for attrition. Thus, the subtraction 1.0 - 0.9 gives the value 0.1 as the attrition rate from job category 1, and so on.

We now turn to the procedure for altering the historical transitions. This additional flexibility, which is wanted, will be provided in a form given by the following expression,

$$\text{Those On Board} = \text{Those Expected to Transit} + (\pm\ \text{Flexibility}) + \text{Hires} - \text{RIF's}$$

where RIF's = Reductions in Force. The first term on the right, i.e., "Those Expected to Transit," involve the matrix, M, of historical transition rates. The second term, i.e., "$\pm$ Flexibility," involves new variables, i.e., the z^k_{ij} and y^k_{ij} from the model to increase or decrease the number of entrants into specific job categories from other specific categories as a result of discretionary action. The sub-matrices corresponding to this flexibility are represented by the T matrices in Figure 1 with entries which are $\pm$ 1 or 0. In a manner consistent with the Merit Promotion System, this makes it possible to achieve what is wanted in altering the projected steady state probabilities from their previous historical values.

This alteration in transit conditions must also allow for STARTING VALUES, which state the number of each type of personnel on-board at each job level, at the point in time before the period actually begins. Given the initial population values for the transit condition rows shown in Figure 1, these possibilities are represented in the identity matrices I, one for the minorities and one for the non-minorities in these rows. For this illustrative example, we simplified matters by assuming that none of those initially on board were in the training category.

The MAXIMUM ADDITIVE FLEXIBILITY constraints, in Figure 1, provide a control over the positive flexibility in the transition rates. Via these constraints, the additional (flexible) transfers of some particular personnel type out of a job category are not allowed to exceed the total number of personnel of that type who were in the category at the start of the time period. Further specifications may also be made, if desired, via policy parameters that stipulate the limits on this flexibility for long-run transitions. These policy parameters only affect upward mobility in job level and permit increases in the transition rates from job category 1 to 2, 1 to 3, and from category 2 to 3 at most at the indicated values, and Figure 4 is derived from Figure 3, accordingly.

Of course lower limits may be similarly provided to these transition alterations via the constraints which are labeled "Max. Subtractive Flexibility," but these are here provided directly in the model so that no additional data are needed.

Figure 4

MAXIMUM ADDITIVE FLEXIBILITY COEFFICIENTS

FROM	JOB LEVEL 1=LOW	JOB LEVEL 2=TRAINING	JOB LEVEL 3=HIGH
	.1+.05=.15	.2+.7=.9	0

Turning next to Min. EEO Proportions, specific numerical values for these $p_i^k(t)\ g_i(t)$, i.e., the "proportional lower bounds", are supplied for this example in Figure 5. Thus, in our case at least, 10% of the total population in job category 1 will be of minority status at the end of the first time period, 15% in training, and so on.

Figure 5

MINIMUM EEO PROPORTIONS

	JOB LEVEL 1=LOW	JOB LEVEL 2=TRAINING	JOB LEVEL 3-HIGH
FIRST TIME PERIOD	.1	.15	.1
SECOND TIME PERIOD	.1	.15	.1

The final constraints in Figure 1 deal with BUDGETARY issues. They are composed of three sets of inequalities The first represents the total salary budget for all job occupants in each time period. Form Figure 6, for our example, we see that job categories 1 and 2 pay the same salary, i.e., $10,000/year, while there is a 50% jump in salary when an individual moves to job category 3. The budget available to cover all salaries is $120,000,000 in each period.

Figure 6

TOTAL SALARY BUDGET

	JOB LEVEL 1=LOW	JOB LEVEL 2=TRAINING	JOB LEVEL 3=HIGH
FIRST TIME PERIOD	10,000	10,000	15,000
SECOND TIME PERIOD	10,000	10,000	15,000

$b^1(1)$ = \$120,000,000
$b^1(2)$ = \$120,000,000

The second set of budget relations in Figure 1 deals with transfer costs (salary plus training) incurred during each time period for the flexible transfers from the previous period. Figure 7 budgets these to maximum totals of $50,000 for each period in our example. From the cells of Figure 7 we can see that transferring from job category 1 to category 2, from category 1

to 3 or from category 2 to 3 all incur the same cost -- viz., $1,000.

Figure 7

TRANSFER COSTS

FROM / TO	JOB LEVEL 1=LOW	JOB LEVEL 2=TRAINING	JOB LEVEL 3=HIGH
JOB LEVEL 1=LOW	0	0	0
JOB LEVEL 2=TRAINING	1,000	0	0
JOB LEVEL 3=HIGH	1,000	1,000	0

$$b^2(1) = \$50,000$$
$$b^2(2) = \$50,000$$

The third set of budget relations cover salaries plus recruiting costs for new hires, or penalties incurred as a result of RIF's. Figure 8, for our example, states all new hire costs at $2,000 to cover costs of recruiting and hiring into both job categories 1 and 3. Similarly, costs of $6,000 cover the RIF's from both categories and the totals for hires and RIF's may not exceed $550,000 in either period.

Figure 8

Hires

	JOB LEVEL 1=LOW	JOB LEVEL 3=HIGH
FIRST TIME PERIOD	2,000	2,000
SECOND TIME PERIOD	2,000	2,000

RIF's

	JOB LEVEL 1=LOW	JOB LEVEL 3=HIGH
FIRST TIME PERIOD	6,000	6,000
SECOND TIME PERIOD	6,000	6,000

$$b^3(1) = \$550,000$$
$$b^3(2) = \$550,000$$

Putting all these data together a solution can be obtained via standard linear programming algorithms. The results of such a solution using the above data are given in Tables 9a through 9e.

We now interpret the results for this hypothetical example as follows: In the first period the total manpower goals are not achieved for job categories 1 and 2 as evidenced by the presence of negative deviations of -3,989 in the solution for the total goal constraint for job category 1, and of -1,125 in the solution for the total goal constraint for job category 2. However, the manpower goals for job category 3 is met as witness the zero deviation for this job category in the same Figure. Continuing with this same Figure, the proportional goals are achieved for job category 1, and surpassed for job category 3, and are not met for job category 2.

Turning now to Figure 9b, flexible transfers are seen to occur for both minority and non-minority personnel, and, see Figure 9a, there are new hires for both types of personnel into job category 1 positions in this period. Budgetary constraints for recruitment are binding, but the salary budget and the transfer budget are slack.

Next via Figure 9c we see that the two period solution is such that the total goals for job categories 2 and 3 are also satisfied in the second time period, and, in fact, the total goal for job category 3 is exceeded; however, once again the category 1 total goal is not met. The minimum proportions are again met in this period, of course, and, in fact, all of the proportional goals are fulfilled with the second and third such goals being exceeded. Thus, in two of the job categories both short-term (operational) and long-term (EEO) considerations are satisfied.

Figure 9a

SOLUTION OF THE EEO MODEL FOR THE 1ST TIME PERIOD

				PROPORTIONAL		TOTAL	
	ABOARD	HIRES	RIF'S	GOAL	DISCREPANCIES	GOAL	DISCREPANCIES
MINORITY IN JOB LEVEL 1	14,000	6,500	0	14,000	0	70,000	-3,989*
NON-MINORITY IN JOB LEVEL 1	52,011	21,000	0				
MINORITY IN JOB LEVEL 2	1,500			2,000	-500	10,000	-1,125**
NON-MINORITY IN JOB LEVEL 2	7,375						
MINORITY IN ~~JOB LEVEL 3~~	1,463	0	0	1,400	+63	7,000	0
NON-MINORITY IN JOB LEVEL 3	5,537	0	0				

* −3,989 = 70,000 − (14,000 + 52,011)

** −1,125 = 10,000 − (1,500 + 7,375)

Figure 9b

Flexible Transfers in 1st Time Period

Minority Personnel

TO \ FROM	JOB LEVEL 1=LOW	JOB LEVEL 2=TRAINING	JOB LEVEL 3=HIGH
JOB LEVEL 1=LOW	0	-275	0
JOB LEVEL 2=TRAINING	500	0	0
JOB LEVEL 3=HIGH	0	0	- 225

Flexible Transfers in 1st Time Period

Non-Minority Personnel

TO \ FROM	JOB LEVEL 1=LOW	JOB LEVEL 2=TRAINING	JOB LEVEL 3=HIGH
JOB LEVEL 1=LOW	0	0	0
JOB LEVEL 2=TRAINING	3,375	0	0
JOB LEVEL 3=HIGH	0	0	-3,375

Figure 9c

SOLUTION OF THE EEO MODEL, FOR THE 2ND TIME PERIOD

	ABOARD	HIRES	RIF'S	PROPORTIONAL GOAL	PROPORTIONAL DISCREPANCIES	TOTAL GOAL	TOTAL DISCREPANCIES
MINORITY IN JOB LEVEL 1	16,000	5,200	0	16,000	0	80,000	-1,217
NON-MINORITY IN JOB LEVEL 1	62,783	22,300	0				
MINORITY IN JOB LEVEL 2	3,500			2,000	+1,500	10,000	0
NON-MINORITY IN JOB LEVEL 2	6,500						
MINORITY IN JOB LEVEL 3	1,750	0	0	1,600	+150	8,000	+5,713
NON-MINORITY IN JOB LEVEL 3	11,963	0	0				

Figure 9d

Flexible Transfers in 2nd. Time Period

Minority Personnel

TO \ FROM	JOB LEVEL 1=LOW	JOB LEVEL 2=TRAINING	JOB LEVEL 3=HIGH
JOB LEVEL 1=LOW	0	0	0
JOB LEVEL 2=TRAINING	2.100	0	0
JOB LEVEL 3=HIGH	0	0	-132

Flexible Transfers in 1st Time Period

Non-Minority Personnel

TO \ FROM	JOB LEVEL 1=LOW	JOB LEVEL 2=TRAINING	JOB LEVEL 3=HIGH
JOB LEVEL 1=LOW	0	0	0
JOB LEVEL 2=TRAINING	1,299	0	0
JOB LEVEL 3=HIGH	0	0	-784

Figure 9e

Expenditures

	TRANSFER	RECRUITMENT ETC.	SALARY
FIRST TIME PERIOD	38,750	550,000	85,385,714
SECOND TIME PERIOD	33,989	550,000	109,352,588

Flexible transfers are evident once more in the second time period for both personnel types, as are new hires in the first job category. See Figure 9d. Finally, turning to the planned expenditure of data drawn together in Figure 9e we see that over this period of time the recruitment expenditure constraint is binding. I.e., the permissible limit of $550,000 noted at the bottom of Figure 8 is achieved. The transfer constraint is not binding, however, (see bottom of Figure 7) and it is once more the case that the salary constraint is easily met (cf. Figure 6).

6. Implementation Possibilities

Of course this is an hypothetical (highly simplified) example and intended only to illustrate the model which has been developed to this point. It should also be noted that anything like a "solution," such as the preceding one, would only be a start for an analysis that would certainly continue into sensitivity testing on other types of validation. After a stage of initial implementation has been readied, moreover, it would be best as far as user involvement is concerned, to proceed by developing interactive computer capabilities. Experience has shown, however, that considerable experience in a batch environment is necessary prior to operational testing via interactive computer techniques.[1] Necessary contacts with all affected areas of management would thereby be facilitated much sooner, considering the developmental time necessary to implement conversational models on an operational basis.

Many problems can be expected prior to any such actual implementation. For instance, the civilian manpower force of the U. S. Navy is spread across numerous activities in many different parts of the country. A question therefore arises whether Equal Employment Opportunity goals should apply to

1/ See [8]

the minority mix of the nation as a whole or only to the minority mixes surrounding individual installations. An answer to this question may require recourse to multi-level modeling procedures in which all aspects -- local, national, etc. -- can be examined simultaneously.

Of course, a further distinction may need to be made between policy (i.e., Navy-wide policy) and its implementation at local, national and other levels in any case. Data synthesis and quality would almost certainly need to be considered and weighted against the ways in which it might be used. Choices of weights and other alterations in the model would require attention, as we indicated at the outset, and this does not exhaust the possibilities either.

These issues are best confronted, we think, by research (basic or otherwise) which is carried out in a context of actual applications and in liaison with officials who are responsible for these programs. The model can provide new and needed assistance[1] for this purpose. It does provide a variety of new and improved possibilities for manpower planning, not only in equal employment opportunities but in other areas as well. Abstract considerations and the numerical illustrations all indicate this, and so do the discussions which have been conducted with the officials responsible for phases of such manpower plans and their implementation.

[1] See [4]

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